# M.Sc. 2nd Semester (2020)

# Measure Theory and Functional Analysis

1.) **Open Set:-** A set $A \subset R$ is called an open set if given,

$x \in A$, $\exists \epsilon > 0$, such that $\epsilon$ - neighbourhood of $x$ is a subset of $A$.

$\therefore$ Every $\epsilon$ - neighbourhood is an open set but an open set is not necessarily an $\epsilon$ - neighbourhood.

**Example:-** (i) The set of all real numbers $x$, such that $0 < x < 2$ is a 1-neighbourhood of the point $x_0 = 1$, as well as this set is an open set.

(ii) The set of all real numbers $x$, such that $0 < x < 5$ is an open set but not an $\epsilon$ - neighbourhood of any $\epsilon > 0$.

(2.) Continuity :→ A real valued function 'f' is said to be continuous at the point $x_0$, if given $\epsilon > 0$, $\exists\, \delta > 0$, such that:

$|f(x) - f(x_0)| < \epsilon$, whenever $|x - x_0| < \delta$.

If a function is continuous at each point $x$ for which it is defined, then it is called a continuous function.

(3.) Limit point and Derived Set :-

A point '$x$' is said to be a limit point of a set 'A', if every neighbourhood 'G' of $x$ contains a point of 'A', other than $x$, i.e.,

'$x$' is said to be a limit point of A, if for every neighbourhood 'G' with $x \in G$,

$$(G - \{x\}) \cap A \neq \phi .$$

(4) Condensation Point :- A point 'x' is said to be a condensation point of a set A, if every neighbourhood containing x contains an infinite number of points of A.

(5) Closed Sets :- A set A is said to be closed if every limiting point of A belongs to the set A itself.

Symbolically, a set A is said to be closed if $D(A) \subset A$.

Examples: (i) $\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \cdots\} = A$, is a closed set, for $D(A) = \{0\} \subset A$.

(ii) A closed interval is a closed set.
For $D([a,b]) = [a,b] \subset [a,b]$.

(iii) Every real number is a limit point so that rational limit point belongs to be set Q.

This $\Rightarrow D(R) = R \subset R \quad D(Q) = R \not\subset Q$

So that R is closed set whereas Q is not.